Discharge Analysis in Dielectric Liquids by Fully Coupled Finite Element Method for Dissociation and Thermal Effects

Ho-Young Lee, Geun-Young Jeong, and Se-Hee Lee
School of Electrical Engineering and Computer Science, Kyungpook National University, Daegu 702-701, Korea
shlees@knu.ac.kr

Abstract — The response of lightning impulse voltage was explored in dielectric liquids employing the hydrodynamic modeling with three charge carriers by using the fully coupled finite element method for analyzing the dissociation and thermal effects. To understand the physical behaviors of discharge phenomena in dielectric liquids, the response of step voltage has been extensively studied recently by using numerical techniques, but that of lightning impulse voltage was rarely seen in technical literatures. Especially, the dissociation effect has been usually excluded in the discharge simulation for dielectric liquids. We, here, tested the impulse response with a tip-sphere electrode and compared the experimental results. Finally, we found that the velocity from the numerical result corresponded to that from the experimental result of lightning impulse breakdown testing in the literatures and temperature was significantly increased with the dissociation effect.

I. Hydrodynamic Governing Equations for Streamer Propagation in Dielectric Liquids

Recently, the hydrodynamic drift-diffusion equations combined with the Poisson's equation, which were employed here, is widely adopted to simulate the discharge analysis in dielectric liquids such as transformer oils as follows [1]-[3]:

$$-\nabla\cdot(\varepsilon\nabla V)=\rho_+ +\rho_- +\rho_e \tag{1}$$

$$\frac{\partial\rho_+}{\partial t}+\nabla\cdot(\mathbf{J}_+)=G_I(|\mathbf{E}|)+ecK_D^0F(|\mathbf{E}|)+\frac{\rho_+\rho_eR_{+e}}{e}+\frac{\rho_+\rho_-R_{+-}}{e} \tag{2}$$

$$\frac{\partial\rho_e}{\partial t}+\nabla\cdot(\mathbf{J}_e)=-G_I(|\mathbf{E}|)-\frac{\rho_+\rho_eR_{+e}}{e}-\frac{\rho_e}{\tau_a} \tag{3}$$

$$\frac{\partial\rho_-}{\partial t}+\nabla\cdot(\mathbf{J}_-)=\frac{\rho_e}{\tau_a}-ecK_D^0F(|\mathbf{E}|)-\frac{\rho_+\rho_-R_{+-}}{e} \tag{4}$$

$$\rho_l c_v\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k_T\nabla^2T+\mathbf{E}\cdot\mathbf{J} \tag{5}$$

where the subscript +, −, and e represent the positive, negative ions, and electron, respectively, ε the dielectric permittivity, V the electric scalar potential, ρ the charge density, t the time, μ the mobility, $\mathbf{E}$ the electric field intensity, $G_I(|\mathbf{E}|)$ the electric field dependent molecular ionization source term, e the electron charge, cK_D^0 the zero field equilibrium value for the number density rate, $F(|\mathbf{E}|)$ the dissociation constant's electric field enhancement, R_{xy} the recombination rate of x and y carriers, τ_a the electron attachment time constant, ρ_l the oil's density, c_v the oil's specific heat, T the temperature of the liquid in Kelvin, $\mathbf{v}$ the oil's velocity, k_T the oil's thermal conductivity, and $\mathbf{E}\cdot\mathbf{J}$ the dissipative source term. The electric field dependent molecular ionization can be expressed as [1]

$$G_I(|\mathbf{E}|)=\frac{e^2n_0a}{h}|\mathbf{E}|\exp\left(-\frac{\pi^2m^*a\Delta^2}{eh^2|\mathbf{E}|}\right) \tag{6}$$

where e is the electron charge, the number density of ionizable molecules n_0=1 x 10^{21} m^{-3}, the molecular separation constant a=3 x 10^{-10} m, h the Plank's constant, the effective electron mass in the dielectric liquid m^*=9.1 x 10^{-32} kg, and the molecular ionization energy Δ=7.1 eV. The dissociation constant's electric field enhancement can be briefly expressed as [2]

$$F(|\mathbf{E}|)=\frac{I_1(4b)}{2b} \tag{7}$$
$$\text{with } b=\sqrt{\frac{e^3|\mathbf{E}|}{16\pi\varepsilon k^2T^2}}$$

where I_1 is the modified Bessel function of the first kind, k the Boltzmann's constant, and T the temperature of the insulating liquid.

To consider the charge injection mechanism, the Fowler-Nordheim field emission and Richardson-Dushman thermionic emission processes were adopted on the cathode. When the strong electric field intensity was applied between the gap, the electron will be injected into the discharge space and forming the initiation of streamer.

II. Waveform of Lightning Impulse Voltage

To test the effects of lightning impulse voltage, we used a source of lightning voltage with 1.2/50μs as shown in Fig. 1. A convenient analytical representation of the pulse waveform is the double-exponential expression used here as

$$V(t)=V_1[\mathrm{Exp}(-t/\tau_2)-\mathrm{Exp}(-t/\tau_1)] \tag{8}$$
$$\text{with } \tau_2=T_t/\ln(2),\ \tau_1=T_f/5,\ V_1=V_m\mathrm{Exp}(T_f/1.443T_t)$$

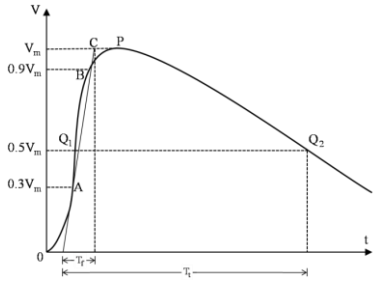
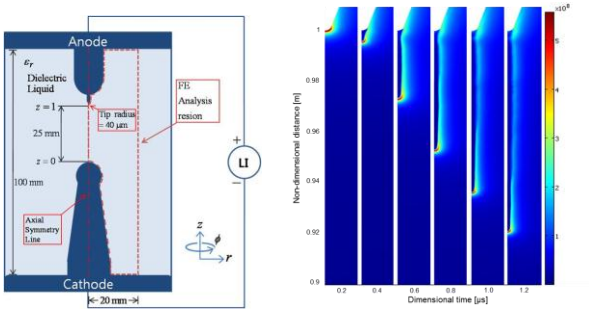


Fig.1. Waveform of lightning impulse for numerical simulation. The rising time, T_f, is 1.2μs and the 50% of falling time, T_t, is 50μs. V_m denotes the maximum peak voltage.

III. Numerical Results

The tip-sphere electrodes plotted in Fig. 2(a) introduced in the IEC standard #60897 was tested for analyzing the discharge phenomena in a dielectric liquid with the lightning impulse. Fig. 2(b) shows the propagation of electric field wave with time and the order of initial electric field intensity around the head of tip was 10^8 V/m, which is sufficient for initiation of streamer propagation [1]. Figs. 3 to 5 show the temporal dynamics of electric field intensity, electric scalar potential, ions, electron charge and space charge densities with time along with the axial axis, respectively. The average speed of electric field wave was approximately 1.5 km/s from this numerical analysis, which is almost the same as the experimental results from [4]. Experimentally, the velocity of positive streamer was 1.59 km/s with the same geometry [4]. The final temperature distribution was significantly increased with dissociation effect as shown in Fig. 6.



(a) Analysis model (b) Temporal electric field distribution

Fig. 2. Numerical analysis model and propagations of electric field wave with specific times as shown in Fig. 1.

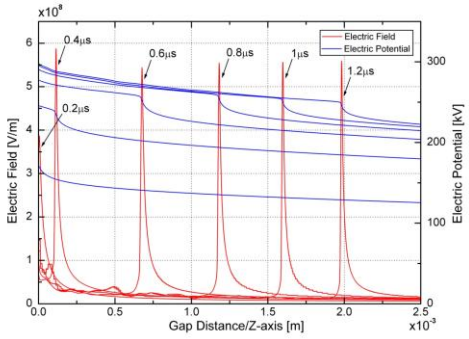


Fig. 3. Distributions of electric field and electric potential with time.

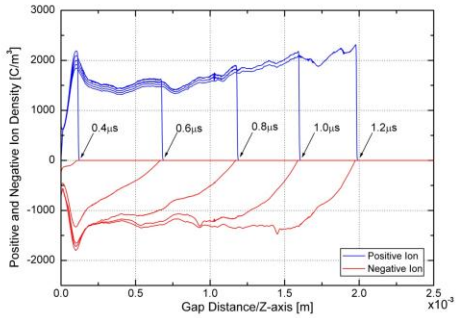


Fig. 4. Distributions of positive and negative ions with time.

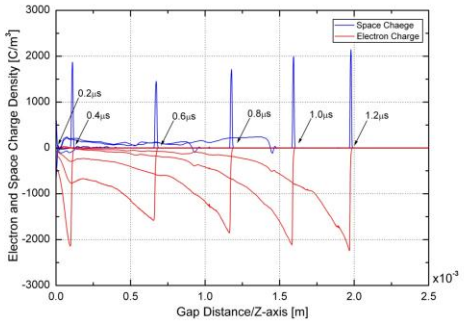


Fig. 5. Distributions of space and electron charge densities with time.

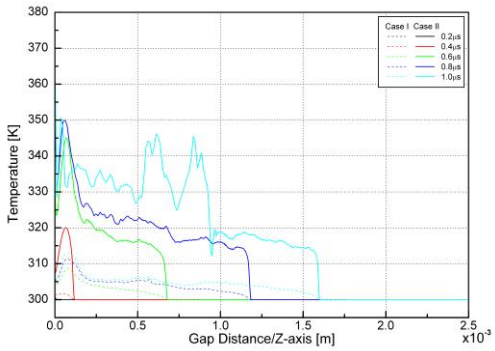


Fig. 6. Distributions of temperature with time. The 'Case I' denotes the previous studied result excluded the dissociation and the 'Case II' denotes the newly developed result including the dissociation.

IV. References

[1] F. M. O'Sullivan, "A Model for the Initiation and Propagation of Electrical Streamers in Transformer Oil and Transformer Oil Based Nanofluids," Ph.D dissertation, Massachusetts Institute of Technology, Cambridge, MA, USA, 2007.

[2] F. O'Sullivan, S. H. Lee, M. Zahn, "Modeling the Effect of Ionic Dissociation on Charge Transport in Transformer Oil," *IEEE conf. Elec. Insul. Diel. Phen.*, pp. 756-759, Kansas City, Missouri, USA, October, 2006.

[3] J. George Hwang, et al., "Effects of Nanoparticle Charging on Streamer Development in Transformer Oil Based Nanofluids," *J. Appl. Phys.*, 107, No. 1, 014310, 2010.

[4] V. Segal, et al., "AC(60Hz) and Impulse Breakdown Strength of a Colloidal Fluid Based on Transformer Oil and Magnetite Nanoparticles," *IEEE Int. Symp. On Elec. Insul.*, Arlington, Virginia, USA, June 1998.